COSMOLOGICAL CONSTANT AS TOPOLOGICAL INVARIANT

Paweł Klimasara[1]

(in collaboration with Jerzy Król[1], Krzysztof Bielas[1], and Torsten Asselmeyer-Maluga[2])

[1]University of Silesia in Katowice
Katowice, Poland

[2]German Aerospace Center
Berlin, Germany

The large-scale smoothness of the Universe

Does it need to be standard smoothness?

- Mathematical ambiguity

Exotic $\mathcal{R}^n$ - a differentiable manifold that is homeomorphic but not diffeomorphic to standard smooth $\mathbb{R}^n$

- It doesn't exist for all $n \neq 4$.
- In fourth dimension there are uncountably many (non-isomorphic ones)!

All Exotic $\mathcal{R}^4$'s have necessarily non-vanishing Riemann curvature

A PHYSICAL MEANING?

Cosmological Constant value as a topological invariant

But: Curvature depends crucially on embeddings

Our Strategy

Embedding: (small exotic $\mathcal{R}^4$) $\longrightarrow \mathbb{R}^4 \longrightarrow M^4$

- $\star$ invariant part of the scalar curvature

Is it even possible that appropriate M^4 exists?

Yes!

$$K3\#\overline{CP(2)}$$

Embedding (small exotic $\mathcal{R}^4$) $\longrightarrow K3\#\overline{CP(2)}$
is allowed by two topology changes
$\mathcal{S}^3 \longrightarrow \Sigma(2,5,7) \longrightarrow P\#P$

Topological calculations versus Planck data

$$\Omega_\Lambda = \frac{c^5}{24\pi^2 hGH_0^2} \cdot \exp\left(-\frac{3}{CS(\Sigma(2,5,7))} - \frac{3}{CS(P\#P)} - \frac{\chi(A_{\text{cork}})}{4}\right)$$

$$\Omega_\Lambda \approx 0.7029$$

$$(\Omega_\Lambda)_{\text{PLANCK}} = 0.6911 \pm 0.0062$$

Could the large-scale smoothness be deduced from—or connected with the initial quantum state?

$\mathcal{H}$ - a separated complex Hilbert space

Lattice of Projections $\mathbb{L} = (\mathbb{L}(\mathcal{H}), \vee, \wedge)$

$\mathbb{L}(\mathcal{H})$ contains projections on closed linear subspaces of $\mathcal{H}$

$$P_1 \wedge P_2 \equiv P_1 \cap P_2$$
$$P_1 \vee P_2 \equiv \text{span}(P_1 \cup P_2)$$

$\longrightarrow$ partially ordered by set inclusion

$\star$ Spectral theorem and Boolean subalgebras of $\mathbb{L}(\mathcal{H})$

Let $\mathcal{H} = L^2(\mathbb{R}^n, d^n x)$ and B_Q be the complete maximal Boolean subalgebra of $\mathbb{L}$ containing position operator $\hat{Q}$ on $\mathcal{H}$.

$\star$ MODELS OF ZFC SET THEORY

$B_Q \backsimeq Bor(\mathbb{R}^n)/_{\mathcal{N}} \longrightarrow$ RANDOM FORCING

$\star$ REAL NUMBERS IN $Sh(B_Q)$

EXTENSION OF THE REAL LINE IN RANDOM FORCING:

$$R \longrightarrow R[G],\ R \subset R[G]$$

PHYSICS IN LARGE SCALES:

$\mathbb{R}$ AS DEDEKIND COMPLETE ORDERED FIELD

THE GAP

(1st order) $R, R[G] \Longrightarrow \mathbb{R}$ (2nd order)

$\star$ THEY EXIST ONLY FOR $n = 4 \rightarrow$ 4D SPACETIME

Canceling of zero modes

The zero-point energy of quantum field corresponding to a particle of mass m:

$$\int_{R^3} \frac{d^3k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2+m^2}}{2}, \; m \in R[G], \; \mathbf{k} \in R^3.$$

Here all such integrals equal 0 since we integrate over the null set $R^3 \subset R^3[G]$.

QM lattice of projections:

- vanishing of zero-modes
- necessity of exotic smoothness

The Complete Picture

[1] Król, Asselmeyer-Maluga, Bielas, Klimasara, *From Quantum to Cosmological Regime. The Role of Forcing and Exotic 4-Smoothness*. Universe 2017, 3, 31.
[2] Asselmeyer-Maluga, Torsten, and Jerzy Król. *How to obtain a cosmological constant from small exotic R4.* Physics of the dark universe 19 (2018): 66-77.

Low-Dimensional Topology:

- value of cosmological constant
- necessity of four dimensions

~ Thank You ~
for Your Attention